Is the cosmological "constant" a nonlocal quantum residue of discreteness of the causal set type?*

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Abstract

The evidence for an accelerating Hubble expansion appears to have confirmed the heuristic prediction, from causal set theory, of a fluctuating and "ever-present" cosmological term in the Einstein equations. A more concrete phenomenological model incorporating this prediction has been devised and tested, but it remains incomplete. I will review these developments and also mention a possible consequence for the dimensionality of spacetime.

The inference from causet theory of a fluctuating cosmological constant Λ is possibly the earliest theoretical prediction of a non-zero Λ; and yet relatively little work has been devoted to developing it. In its original form [1], the prediction yielded only an order of magnitude estimate for Λ, namely that its current value should be about $\pm 10^{-120}$ in Planck units. When evidence started accumulating for a Λ of just this size, it seemed time to try to embed the original prediction in a more complete model. The resulting model, as elaborated by Scott Dodelson, Patrick Greene, Maqbool Ahmed and me, had some ad hoc elements, but it realized concretely the original implication of an "ever-present Λ", that is one whose (fluctuating) magnitude is always comparable to the Hubble scale. [2] [3]

My talk today will review these developments in the hope that people will be encouraged to carry the underlying idea further. I will explain where the original heuristic prediction came from, and I will describe a concrete model that arose from it. Before doing so, however I want to mention the two greatest weaknesses that the model had.

The first weakness was a certain arbitrariness in how one interprets the idea of a varying Λ. The second was the imposition of spatial homogeneity and isotropy on the model, ie the assumption that the metric takes the FRW form. The first weakness has

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been largely overcome in subsequent work by Maqbool Ahmed but the second still needs to be tackled.

Lambda forgotten

The cosmological constant originated as a non-solution to a non-problem, *viz* the fact that the Einstein equation

$$\frac{1}{\kappa}G^{ab} + \Lambda g^{ab} = T^{ab} \tag{1}$$

does not admit a static cosmos as a solution if the Λ-term is omitted. This, of course, was not a real problem because the cosmos is not static. Nor does the inclusion of Λ solve this non-problem, because the resulting static cosmos is unstable to collapse or unbounded expansion, as is well known. In view of this inauspicious beginning, the introduction of Λ must have seemed unmotivated, and the prejudice grew up among cosmologists that Λ was 0. With few exceptions, they either set $\Lambda = 0$, or they never even bothered to mention it at all, despite many indications from quantum field theory and quantum gravity that things could not possibly be that simple.

Lambda remembered

Within the last decade or so, all that has changed, and the origin of the so called "dark energy" is recognized as a central question of astronomy. Although many lines of thought and observation seem to have contributed to the change (including, for example, the problem that without Λ (or with $\Lambda < 0$), the cosmos seemed to be younger than some of its contents), the two most persuasive arguments concerned the CMB relic radiation* and the observations of distant supernovas. Let us briefly recall both of these arguments.

Spatial flatness and missing mass

As I understand it, this argument proceeds from the angular size of the so called "acoustic peaks" in the CMB to the spatial flatness of the universe (in our vicinity) to the need for a new term in the Einstein equation. The density and temperature of the medium that emitted the CMB photons resulted from the initial gravitational collapse and subsequent fluid oscillation of that medium. This process produced bright regions of approximately known diameter and distance from us, which correspond to the peaks one sees in plots of CMB brightness vs. angular size. But since the *apparent* angular size of a distant object is greater in spherical space than in flat space (and correspondingly less in hyperbolic space), we can infer the radius of curvature of our cosmos, assuming it to be spatially homogeneous

* CMB = cosmic microwave background radiation. It was already detected as early as 1940 [4], but its significance was not appreciated, and it was forgotten.

and isotropic (Friedmann cosmos). The best fit is to zero curvature, i.e. to spatial flatness. But for such a cosmos, the $a = b = 0$ component of eq. (1) reads, if we take $\Lambda = 0$,

$$3\left(\frac{\dot{a}}{a}\right)^2 = \rho \tag{2}$$

where $\rho = T^{00}$ is the total energy density in matter (including the so-called dark matter inferred from galactic rotation curves and gravitational lensing). Also, a is the scale-factor or "radius of the universe", $\dot{a} = da/d\tau$ with $\tau =$ proper time, and I've set $\kappa \equiv 8\pi G = 1$. By comparing this equality with various, more or less direct measurements of ρ, one finds that the latter would have to be tripled in order to satisfy (2). With Λ restored on the other hand, one obtains instead of (2) the equation

$$3\left(\frac{\dot{a}}{a}\right)^2 - \Lambda = \rho \tag{3}$$

from which Λ can be determined once ρ and $H = \dot{a}/a$ are known. The conclusion seems to be that either Λ is nonzero or something rather like it is out there, carrying twice the effective mass-density of non-gravitational matter.

Dim distant supernovas

Observations of supernovas (of type IA) have yielded the most direct evidence for a positive Λ, because they let us deduce the value of the "acceleration" $\ddot{a}$ from a plot of luminosity vs. redshift, and the analysis depends only on the well understood behavior of electromagnetic fields in curved spacetime. This yields for the "luminosity distance" or normalized "dimness" d_L of a known source that is not too far away, the equation

$$Hd_L = z + \frac{1}{2}\left(1 + \frac{a\ddot{a}}{\dot{a}\dot{a}}\right) z^2 + O(z^3) \tag{4}$$

where $H = \dot{a}/a$ is the Hubble constant and z the redshift of the light from the supernova. Clearly, one can deduce both $\dot{a}$ and $\ddot{a}$ if one knows how d_L varies with z. Moreover one sees that, other things being equal, a larger $\ddot{a}$ will produce an image that is *dimmer* at equal redshift. The graph of d_L as a function of z thus shifts upward, and this is what has been seen.

Actually, the sign of this effect can be deduced from the equivalence principle, using only what we know of the Doppler shift in flat spacetime. According to the equivalence principle, spacetime *is* flat in our neighborhood, whence what is usually described as the expansion of space we can reinterpret locally as the flow of galaxies away from us. In such a coordinate system, two supernovas of equal brightness are at equal spatial distance from us. Hence their light was emitted at the same time t_0 in the past. Now let these two stars be in two different spacetimes with the same expansion rate H but different accelerations $\ddot{a}$. In the cosmos with *greater* acceleration, the expansion rate at t_0 was smaller, whence the recession rate of the supernova was smaller, whence its redshift z was also less. Equivalently, the supernova appears dimmer at the same redshift.

To complete the argument, observe that a positive Λ serves to increase $\ddot{a}$. This follows from the $a = b = 1$ component of (1), and can be remembered from the fact that, for pure $\Lambda > 0$, one obtains a de Sitter cosmos, in which a grows exponentially with proper time τ. By contrast, in a cosmos with $\Lambda = 0$, the expansion necessarily slows down, responding to the gravitational attraction of the galaxies for each other.

The Λ puzzle

The evidence we have just reviewed points to a cosmological constant of magnitude, $\Lambda \approx 10^{-120}\kappa^{-2}$, and this raises two puzzles:† Why is Λ so small without vanishing entirely, and Why is it so near to the critical density $\rho_{critical} = 3H^2$ that appears in eqs. (2) and (3)? Is the latter just a momentary occurrence in the history of the universe (which we are lucky enough to witness), or has it a deeper meaning?

Clearly both puzzles would be resolved if we had reason to believe that $\Lambda \approx H^2$ always. In that case, the smallness of Λ today would merely reflect the large age of the cosmos. But such a Λ would conflict with our present understanding of nucleosynthesis in the early universe and of "structure formation" more recently. (In the first case, the problem is that the cosmic expansion rate influences the speed with which the temperature falls through the "window" for synthesizing the light nuclei, and thereby affects their abundances. According to (3) a positive Λ at that time would have increased the expansion rate, which however is already somewhat too big to match the observed abundances. In the second case, the problem is that a more rapid expansion during the time of structure formation would tend to oppose the enhancement of density perturbations due to gravitational attraction, making it difficult for galaxies to form.) But neither of these reasons excludes a *fluctuating* Λ with typical magnitude $|\Lambda| \sim H^2$ but mean value $\langle\Lambda\rangle = 0$. The point now is that *such fluctuations can arise as a residual, nonlocal quantum effect of discreteness*, and specifically of the type of discreteness embodied in the causal set.

Features of causet theory needed in the following

In order to explain this claim, I will need to review some basic aspects of causet theory. [5] According to the causal set hypothesis, the smooth manifold of general relativity dissolves, near the Planck scale, into a discrete structure whose *elements* can be thought of as the

† I prefer the word puzzle or riddle to the word problem, which suggests an inconsistency, rather than merely an unexplained feature of our theoretical picture.

"atoms of spacetime". These atoms can in turn be thought of as representing "births", and as such, they carry a relation of ancestry that mathematically defines a *partial order*, $x \prec y$. Moreover, in our best dynamical models [6], the births happen sequentially in such a way that the number n of elements plays the role of an auxiliary time-parameter. (In symbols, $n \sim t$.)[♭]

Two basic assumptions complete the kinematic part of the story by letting us connect up a causet with a continuum spacetime. One posits first, that the underlying microscopic order $\prec$ corresponds to the macroscopic relation of before and after, and second, that the number of elements N comprising a region of spacetime equals the *volume* of that region in fundamental (i.e. Planckian) units. (In slogan form: geometry = order + number.) The equality between number N and volume V is not precise however, but subject to Poisson fluctuations, whence instead of $N = V$, we can write only

$$N \sim V \pm \sqrt{V} \ . \tag{5}$$

(These fluctuations express a "kinematical randomness" that seems to be forced on the theory by the noncompact character of the Lorentz group.)

To complete the causet story, one must provide a "dynamical law" governing the birth process by which the causet "grows" (the discrete counterpart of equation (1)). This we still lack in its quantum form, but for heuristic purposes we can be guided by the classical sequential growth (CSG) models referred to above; and this is what I have done in identifying n as a kind of time-parameter.

Ever-present Λ

We can now appreciate why one might expect a theory of quantum gravity based on causal sets to lead to a fluctuating cosmological constant. Let us assume that at sufficiently large scales the effective theory of spacetime structure is governed by a gravitational path-integral, which at a deeper level will of course be a sum over causets. That n plays the role of time in this sum suggests that it must be held fixed, which according to (5) corresponds to holding V fixed in the integral over 4-geometries. If we were to fix V exactly, we'd be doing "unimodular gravity", in which setting it is easy to see that V and Λ are conjugate to each other in the same sense as energy and time are conjugate in nonrelativistic quantum

[♭] It is an important constraint on the theory that this auxiliary time-label n should be "pure gauge" to the extent that it fails to be determined by the physical order-relation $\prec$. That is, it must not influence the dynamics, this being the discrete analog of general covariance.

mechanics. [This conjugacy shows up most obviously in the Λ-term in the gravitational action-integral, which is simply

$$-\Lambda \int \sqrt{-g}\, d^4x = -\Lambda V \ . \tag{6}$$

It can also be recognized in canonical formulations of unimodular gravity [7], and in the fact that (owing to (6)) the "wave function" $\Psi({}^3g;\Lambda)$ produced by the unrestricted path-integral with parameter Λ is just the Fourier transform of the wave function $\Psi({}^3g;V)$ produced at fixed V.] In analogy to the $\Delta E \Delta t$ uncertainty relation, we thus expect in quantum gravity to obtain

$$\Delta\Lambda \, \Delta V \sim \hbar \tag{7}$$

Remember now, that even with N held exactly constant, V still fluctuates, following (5), between $N+\sqrt{N}$ and $N-\sqrt{N}$; that is, we have $N \sim V \pm \sqrt{N} \Rightarrow V \sim N \pm \sqrt{V}$, or $\Delta V \sim \sqrt{V}$. In combination with (7), this yields for the fluctuations in Λ the central result

$$\Delta\Lambda \sim V^{-1/2} \tag{8}$$

Finally, let us *assume* that, for reasons still to be discovered, the value about which Λ fluctuates is strictly zero: $\langle\Lambda\rangle = 0$. (This is the part of the Λ puzzle we are *not* trying to solve.*) A rough and ready estimate identifying spacetime volume with the Hubble scale H^{-1} then yields

$$V \sim (H^{-1})^4 \sim H^{-4} \ \Rightarrow \ \Lambda \sim V^{-1/2} \sim H^2 \sim \rho_{critical}$$

(where I've used that $\Lambda = \Lambda - \langle\Lambda\rangle$ since $\langle\Lambda\rangle = 0$). In other words, Λ would be "ever-present" (at least in 3+1 dimensions).

A concrete model incorporating equation (8)

In trying to develop (8) into a more comprehensive model, we not only have to decide exactly which spacetime volume 'V' refers to, we also need to interpret the idea of a varying Λ itself. Ultimately the phenomenological significance of V and Λ would have to be deduced from a fully developed theory of quantum causets, but until such a theory is available, the best we can hope for is a reasonbly plausible scheme which realizes (8) in some recognizable form.

As far as V is concerned, it pretty clearly wants to be the volume to the past of some hypersurface, but which one? If the local notion of "effective Λ at x" makes sense, and if we can identify it with the Λ that occurs in (8), then it seems natural to interpret V as

* But see the ansatz (10) below, which yields $\langle\Lambda\rangle = 0$ automatically.

the volume of the past of x, or equivalently (up to Poisson fluctuations) as the number of causet elements which are ancestors of x:

$$V = \text{volume}(\text{past}(x)) \ .$$

One could imagine other interpretations,[†] but this seems as simple and direct as any.

As far as Λ is concerned, the problems begin with eq. (1) itself, whose divergence implies (at least naively[♭]) that Λ = constant. The model of [2] and [3] addresses this difficulty in two stages. First it limits itself to spacetimes of the FRW form, i.e. it assumes that the cosmos is spatially homogeneous and isotropic (which of course requires for consistency that Λ *also* be spatially homogeneous). Having assumed this, we might as well assume in addition that space is flat ($k = 0$), since that simplifies the equations and matches current data. The Einstein equations (1) then reduce (with $\kappa = 1$) to a pair of ordinary differential equations known as the Friedmann equations:

$$3(\dot{a}/a)^2 = \rho + \rho_\Lambda \tag{9a}$$

$$2\ddot{a}/a + (\dot{a}/a)^2 = -(p + p_\Lambda) \ , \tag{9b}$$

where $\rho_\Lambda \equiv \Lambda$ and $p_\Lambda \equiv -\Lambda$ (corresponding to the familiar "equation of state" of Λ, $p = -\rho$.)

Now in the usual case where Λ is time-independent, equation (9b) is a consequence of (9a); and conversely the two equations are incompatible when $\dot{\Lambda} \neq 0$, this being precisely the difficulty with which we began. To bypass this incompatibility we are forced to modify the Friedmann equations.[⋆] The most straightforward way of doing so is to retain only one of them, or possibly some other linear combination of (9a) and (9b). In reference [2] we followed this approach by adopting (9a) as our "dynamical guide" and discarding (9b). This choice is appealing because the resulting dynamics is easy to simulate, and because it admits an alternative description in which *neither* (9a) nor (9b) is compromised, but instead the "equation of state of Λ" is modified in a simple, local manner. Fortunately, changing one's "guide" by adopting a different linear combination of (9a) and (9b) appears

[†] For example, interpretations in which Λ is not a spacetime field at all, but must be understood more nonlocally.

[♭] Naively because it neglects the circumstance that a fluctuating Λ would be something like a stochastic Brownian function that need not even have a derivative.

[⋆] Notice in this connection that (9b) will not even be well defined if (9a) holds and Λ is a function of Brownian motion type.

to alter nothing qualitatively [3], so let me ignore such possibilities for now. Then our dynamical scheme is just (9) with

$$\begin{aligned} \rho_\Lambda &= \Lambda \\ p_\Lambda &= -\Lambda - \dot{\Lambda}/3H \end{aligned}$$

Finally, to complete our model and obtain a closed system of equations, we need to specify Λ as a (stochastic) function of V, and we need to choose it so that $\Delta\Lambda \sim V^{-\frac{1}{2}}$. But this is actually easy to accomplish, if we begin by observing that (with $\kappa = \hbar = 1$) $\Lambda = S/V \approx S/N$ can be interpreted as the action per causet element that is present even when the spacetime curvature vanishes. (As one might say, it is the action that an element contributes just by virtue of its existence.†) Now imagine that each element contributes (say) $\pm\hbar$ to S, with a random sign. Then S is just the sum of N independent random variables, and we have

$$S/\hbar \sim \pm\sqrt{N} \sim \pm\sqrt{V/\ell^4} \ ,$$

where $\ell \sim \sqrt{\hbar\kappa}$ is the fundamental time/length of the underlying theory, which thereby enters our model as a free phenomenological parameter. This in turn implies, as desired, that

$$\Lambda \ = \ S/V \ \sim \ \pm \frac{\hbar/\ell^2}{\sqrt{V}} \tag{10}$$

We have thus arrived at an ansatz that, while it might not be unique, succeeds in producing the kind of fluctuations we were seeking. Moreover, it lends itself nicely to simulation by computer.

Numerical simulation

Mathematically, our model is defined in the first place by eq. (9a), and secondly by the ansatz for Λ described above, according to which $S = V\Lambda$ is the sum of $N = V/\ell^4$ independent random contributions, where V is the volume of the spacetime region within the past light cone of any point in the hypersurface on which Λ is being evaluated. Strictly speaking, this scheme is not consistently defined since it mixes discrete variables with derivatives; however with $N \sim 10^{240}$ elements currently to our past, we are so close to the continuum that we can safely treat our model as defined by a pair of stochastic differential equations,

$$\frac{da}{a} = \sqrt{\frac{\rho+\Lambda}{3}} d\tau \tag{11}$$

† More properly one should probably think of each element as contributing a multiplicative *phase* $\exp(iS)$. However, our analysis here is only being used to suggest a simple ansatz for Λ, and this ansatz can stand on its own in the present, phenomenological context.

$$V d\Lambda = V d(S/V) = dS - \Lambda \dot{V} d\tau \tag{12}$$

where (11) is just a rewriting of (9a). (Perhaps, though, we should call these integro-differential equations, inasmuch as V, and therefore also the "stochastic driving term" dS, depends on the whole past-history of $a(\tau)$.)

At this point we could try to give our equations a more precise mathematical meaning, but it is just as easy to pass directly to a finite-difference form of them suitable for a computer. I suspect that the scheme described below corresponds to the so-called Itô form of the stochastic system (11)-(12). If so, one might consider also the Stratonovich alternative, but we have not done so.

In references [2] and [3] our model was simulated as follows. Let a_i be the cosmic scale-factor at the ith step and similarly for N_i, V_i, S_i and Λ_i. Let $\rho = \rho_{matter} + \rho_{radition}$, with ρ_{matter} taken to be "dust" scaling like $1/a^3$. Begin at the "Planck time" with the appropriate ratio of $\rho_{matter}/\rho_{radition}$ to end up matching our current universe. Then evolve by iterating the following steps.

- $a_{i+1} = a_i + a_i \sqrt{\frac{\rho_i + \Lambda_i}{3}} (\tau_{i+1} - \tau_i)$
- Given a_{i+1}, compute V_{i+1} using[♭]

$$V(\tau) = \frac{4\pi}{3} \int_0^\tau d\tau' a(\tau')^3 \left(\int_{\tau'}^\tau \frac{d\tau''}{a(\tau'')} \right)^3$$

- $N_{i+1} = V_{i+1}/\ell^4$
- $S_{i+1} = S_i + \xi \sqrt{N_{i+1} - N_i}$ (ξ gaussian with unit variance)
- $\Lambda_{i+1} = S_{i+1}/V_{i+1}$

(The random variable ξ is gaussian thanks to the central limit theorem. It has unit variance because our ansatz arbitrarily took each causet element to contribute $\pm\hbar = \pm 1$ to S. More justified would be $\pm\sigma$ with σ of order unity, but we may omit this new parameter since it coalesces with ℓ in its effect on the model.)

An extensive discussion of the simulations can be found in [3] and [2]. The most important finding was that "tracking behavior" was indeed observed: the absolute value of Λ follows $\rho_{radiation}$ very closely during the era of radiation dominance, and then follows ρ_{matter} when the latter dominates. Secondly, the simulations confirmed that Λ fluctuates

[♭] By rearranging the formulas, one can avoid recomputing this whole integral at each iteration.

with a "coherence time" which is $O(1)$ relative to the Hubble scale. Thirdly, a range of present-day values of Ω_Λ is produced, and these are $O(1)$ when $\ell^2 = O(\kappa)$. (Notice in this connection that the variable Λ of our model cannot simply be equated to the observational parameter Λ^{obs} that gets reported on the basis of supernova observations, for example, because Λ^{obs} results from a fit to the data that presupposes a constant Λ, or if not constant then a deterministically evolving Λ with a simple "equation of state". It turns out that correcting for this tends to make large values of Ω_Λ more likely [3].) Fourthly, the Λ-fluctuations affect the age of the cosmos (and the horizon size), but not too dramatically. In fact they tend to increase it more often than not. Finally, the choice of (9a) for our specific model seems to be "structurally stable" in the sense that the results remain qualitatively unchanged if one replaces (9a) by some linear combination thereof with (9b), as discussed above.

I should emphasize though, that all these results come from simulations with the free parameter ℓ closer to 3 than to 1 (ie to $\sqrt{\kappa}$). If one lowers ℓ much beyond this, the negative fluctuations in Λ typically grow so large that the simulation cannot continue long enough for the cosmos to reach its present size. (The square root in (11) becomes imaginary. In the "linear combination" version of the model, the evolution can continue, but the cosmos recollapses to a singularity.) This creates a tension in the model between the need for the cosmos to reach its present size and the need for the present value of $\Omega_\Lambda = \Lambda/3H^2$ to be as big as it seems to be.

Summary and Outlook

Heuristic reasoning rooted in the basic hypotheses of causal set theory predicted $\Lambda \sim \pm 1/\sqrt{V}$, in agreement with current data. But a fuller understanding of this prediction awaits the "new QCD" ("quantum causet dynamics"). Meanwhile, a reasonably coherent phenomenological model exists, based on simple general arguments. It is broadly consistent with observations but a fuller comparison is needed. It solves the "why now" problem: Λ is "ever-present". It predicts further that $p_\Lambda \neq -\rho_\Lambda$ ($w \neq -1$) and that Λ has probably changed its sign many times in the past.* The model contains a single free parameter of order unity that must be neither too big nor too small.† In principle the value of this parameter is calculable, but for now it can only be set by hand.

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* It also tends to favor the existence of something, say a "sterile neutrino", to supplement the energy density at nucleosynthesis time. Otherwise, we might have to assume that Ω_Λ had fluctuated to an unusually small value at that time. It also carries the implication that "large extra dimensions" will not be observed at the LHC [8].

† unless we want to try to make sense of imaginary time (= quantum tunneling?) or to introduce new effects to keep the right hand side of (9a) positive (production of gravitational waves? onset of large-scale spatial curvature or "buckling"?).

In this connection, it's intriguing that there exists an analog condensed matter system the "fluid membrane", whose analogous parameter is not only calculable in principle from known physics, but might also be measurable in the laboratory! [9]

That our model so far presupposes spatial homogeneity and isotropy is no doubt its weakest feature. Indeed, the ansatz on which it is based strongly suggests a generalization such that Λ-fluctuations in "causally disconnected" regions would be independent of each other; and in such a generalization, spatial inhomogeneities would inevitably arise. Such inhomogeneities were a source of worry in [2] and their potential to disagree badly with the isotropy of the CMB brightness has recently been emphasized in [10] and [11]. On the other hand they could also act as a new type of source for density fluctuations. Without a generalized model allowing for spatial inhomogeneities, one cannot do better than guessing. Let me just note that such a model would evidently have to replace the Einstein equation by some sort of "stochastic PDE", just as our homogeneous model led to a stochastic form of the Friedmann equations.

Closing remarks

In itself the smallness of Λ is a riddle and not a problem. But in a fundamentally discrete theory, recovery of the continuum *is* a problem, and I think that the solution of this problem will also explain the smallness of Λ. (The reason is that if Λ were to take its "natural", Planckian value, the radius of curvature of spacetime would also be Planckian, but in a discrete theory such a spacetime could no more make sense than a sound wave with a wavelength smaller than the size of an atom. Therefore the only kind of spacetime that can emerge from a causet or other discrete structure is one with $\Lambda \ll 1$.) One can also give good reasons why the emergence of a manifold from a causet must rely on some form of nonlocality. The size of Λ should also be determined nonlocally then, and this is precisely the kind of idea realized in the above model.

One pretty consequence of this kind of nonlocality is a certain restoration of symmetry between the very small and the very big. Normally, we think of G (gravity) as important on large scales, with $\hbar$ (quantum) important on small ones. But we also expect that on still smaller scales G regains its importance once again and shares it with $\hbar$ (quantum gravity). If the concept of an ever-present Λ is correct then symmetry is restored, because $\hbar$ rejoins G on the largest scales in connection with the cosmological constant.

Finally, let me mention a "fine tuning" that our model has not done away with, namely the tuning of the spacetime dimension to $d = 4$. In any other dimension but 4, Λ could not be "ever-present", or rather it could not remain in balance with matter. Instead, the same crude estimates that above led us to expect $\Lambda \sim H^2$, lead us in other dimensions to expect either matter dominance ($d > 4$) or Λ-dominance ($d < 4$). Could this be a dynamical reason favoring $3 + 1$ as the number of noncompact dimensions? [12] [10]

A last word

The cosmological constant is just as constant as Hubble's constant.

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