Homework set 1, PHY 790, due Thursday, February 7, 2013

1. Thinking about the material we have covered so far (in the lectures, homework, or reading), write down a physics *question* which you would like to know the answer to. (Some examples: a simple question suggested by the lectures but not answered by them; a more extensive question you think might make a worthwhile research project; a confusion about the material that you'd like to clear up for yourself; a philosophical puzzle related to the material.)

2. In class we had for a spherical dust-filled universe, the action-integral

$$S = \int d\tau(-M + \beta a) - \beta \int \frac{a(da)^2}{d\tau} - 2\pi^2\Lambda \int a^3 d\tau \,,$$

where a is the spatial radius of the universe, M the total rest mass of the dust, $d\tau$ the element of cosmic proper time, and $\beta = 6\pi^2/\kappa$. By varying $d\tau$, we derived an equation of motion that took the form of a "conservation law" also known as the "Hamiltonian constraint". However, we didn't derive the equation of motion that results from varying a. (a) Show that this second equation of motion yields nothing new, beyond what the first one implies. (b) Oops! The previous statement fails in one special case. What is this case and what is the new information? (Notice the analogy with a ball rolling in a one-dimensional potential.)

3. For the dust-filled, spherical Friedmann cosmos, classify the qualitatively different types of cosmological solutions which arise when the parameters M and Λ vary throughout their allowable ranges, $0 \leq M < \infty$, $-\infty < \Lambda < \infty$. (For example, there are solutions in which the universe expands from zero radius and recontracts, solutions in which it stays static at a constant radius, etc.)

4. (a) In class, we expressed the action-functional S for the spherical Friedmann cosmos in terms of the variables a and $d\tau$. Re-express it in terms of the *spatial* 3-volume $v = 2\pi^2 a^3$ and the accumulated *spacetime* 4-volume $T = {}^4V$, which can be defined by the differential relationship $dT = v\, d\tau$. Show that, in terms of v and T, S (or more precisely *minus* S) takes the form of the action-functional for a 1d nonrelativistic particle, with T playing the role of time. What is the mass of this fictitious particle?

(b) Derive the equation of motion for this fictitious particle and show that the cosmological constant Λ can be interpreted as its "total energy". What then is the potential in which the particle is moving?

(c) Taking seriously the analogy with particle-mechanics, consider the variational principle for which T is held fixed at the endpoints. Show that in this case Λ becomes a free constant of integration. (The resulting theory is called "unimodular gravity".)