1 Outline

My research is concerned with a fundamental question: what types of collective behavior are possible in a quantum many-body system? Traditionally, condensed matter theorists have wanted to explain phenomena like magnetism or superconductivity in quantum materials. A new point of view is that *all* that is important in physics is some kind of collective behavior. This means that also the laws that describe computation, or quantum fields, or black hole physics, to name a few, are emergent from the collective behavior of a quantum many-body system.
Some of the questions that I have been asking myself in this spirit are the following: What kind of memory is possible in a quantum many-body system? Does it have to be classical, or can it be quantum? What kind of organization makes quantum memory possible? Are the maximum speed of signals and Lorentz symmetries emergent phenomena? How can we characterize and classify Topological Order? How can thermalization emerge in a closed quantum system?
On the more traditional side, I have been working on developing methods to study Quantum Phase Transitions, the study of Decoherence in spin systems, and the problem of Adiabatic Continuity in many-body systems.
My research combines condensed matter physics, quantum information theory, and mathematical physics.

2 Past Accomplishments

2.1 Topological phases and Entanglement

Since my doctoral studies, I have become fascinated with novel quantum phases of matter, like Topologically Ordered states, like fractional quantum Hall (FQH) liquids or quantum double models. These phases cannot be described in terms of local order parameters and breaking of symmetry [1]. This means that topological states can be found in different phases without a change of symmetry. We were used to the fact that symmetry was the only relevant concept to describe phases of matter. In some sense, normal phases of the matter are classical, in that they can be described by states that are a *product state* in the local degrees of freedom. On the other hand, topological phases cannot be described by product states, but they are

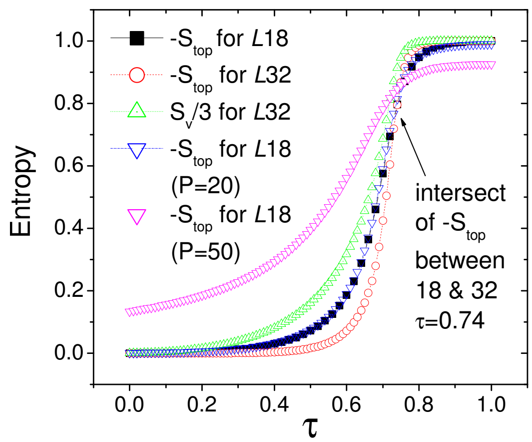

Figure 1: The transition between a normal disordered phase and a topologically ordered phase. The topological entropy is constant $S_{top} = 1$ in the topologically ordered phase $\tau > .74$ and it is thus a universal property of the phase.

always entangled. During my doctorate, I did the first calculation of entanglement in a topologically ordered state [26], namely the toric code model [4]. It turns out that topological phases can be characterized by patterns of long-range entanglement [26, 25, 23, 10, 7] resulting in subleading universal corrections in the entanglement entropy. To what extent entanglement (and entanglement spectrum) can classify topological order was discussed in [7].

Following up on these works, it was shown that indeed such *topological entropy* can distinguish all topologically ordered states from normal states of the matter [4, 6]. The numerical analysis of [20], showed that topological entropy is indeed a universal quantity labeling the topological phase, see Fig.1. In [19], I have shown how the *fidelity metric approach*, i.e., a measure of the susceptibility of the change of the ground state wavefunction under a small change in the parameters of the Hamiltonian, is effective to obtain phase diagrams of topological ordered systems and to extract universal properties of the quantum critical point.

One of the most important problems in the study of topological phases is that of their stability in presence of disorder and at finite temperature. If topological order is lost at any finite temperature it would mean that there is no way to experimentally detect it. As usual, dimensionality plays an important role for the existence of a critical temperature. The only known example with a critical temperature was the toric code in 4D, until in [13] I showed the first example of thermally robust topological order in less than four dimensions.

2.2 Entanglement, thermalization, and quantum memory

Out of equilibrium quantum many-body systems will relax to a steady state when interacting with an environment, just like classical systems. Relaxation consists in some leakage of information from the system to the environment. So while classical states leak classical information, quantum states leak quantum information encoded in the phase relations of the wavefunction. Being so much richer, they also have much more to lose in the process of relaxation. The decoherence process towards the environment is an extremely fast process that is responsible for the appearance of the classical world. The approach to equilibrium of quantum many body systems is of great importance for manifold reasons: **(2.2.1)** practical applications to quantum information processing, **(2.2.2)** understanding of quantum phases, and **(2.2.3)** foundations of quantum statistical mechanics.

2.2.1 Quantum information processing

Storing classical information, using magnetic, mechanical and optical devices, is key to much of everyday lifes technology. These “classical memories” function on the basis that the time scale needed to degrade the information scales with the system size, so in practice memory lasts ‘forever’ even for fairly miniaturized devices.

One of the problems in harvesting the power of quantum information processing is how to realize a stable quantum memory. In [13], I have addressed this question in the case of topologically ordered systems, where information is stored collectively in a many-body state, and only the motion of thermal defects across distances comparable to the system size degrades the information. A stable topological quantum memory can be achieved by appropriately coupling the defects to acoustic waves. They acquire an effective attractive interaction akin to two-dimensional gravity. Below some temperature the system undergoes a gravitational collapse, and the time scale for defects to travel across the system becomes polynomial in system size. This work has been selected as Editor’s choice in Physical Review B.

Another route for a stable quantum memory, is the existence of *decoherence free subspaces*, subspaces of the Hilbert space in which the evolution is decoupled by that of the environment. Unfortunately, such subspaces are unstable. By studying the entanglement dynamics for an open system, I discovered that there are also approximate decoherence free subspaces which can be more robust and viable for experimental implementation [17].

2.2.2 Quantum systems away from equilibrium

The understanding of the non-equilibrium behavior of a quantum system is crucial to understand the universal properties of a many-body ground state wavefunction. This scenario is investigated in the setting of the *quatum quench*, namely a sudden change of the interaction Hamiltonian. In [28], I showed that the mechanism of revivals after a quantum quench is governed by the maximum speed of quasi particles in the system. I studied the quantum quench of topological order in [8]. When looking at some local observables, one can see local thermalization, but non vanishing topological entropy. The work in [8] is mainly numerical and this limits the results to very small systems. In my current research, I am working on finding analytical results for the quantum quench of topological order. We conjecture that topological order is preserved during the evolution in the sense that the topological entropy is conserved during the evolution. This implies that although macroscopic (local) observables will thermalize, there are topological quantities which retain memory of the initial state of the system: this would constitute an example of *quantum glass* and open the way to the study of novel quantum materials.

2.2.3 Foundations of Statistical Mechanics

In the past few years, there has been a strong revival of the interest in the foundations of quantum statistical mechanics. The seminal work of [14] has shown that for a large quantum system that is evolving unitarily, a small part of the system is relaxing towards thermal equilibrium as an effect of the entanglement between the reduced system and the rest. This happens because typical entanglement is almost maximal. The problem with this approach is that random states are not physical because they are not accessible in nature. One needs a doubly exponential time in the size of the system to obtain the statistics of random states. Physical states, are those obtained by evolution with a local Hamiltonian for a reasonable amount of time, namely a time scaling with the size of the system. If physical states are not typical, can they still have typical entanglement? We need to answer this question to know whether we can justify the quantum approach to the foundations of statistical mechanics. This was a question that haunted me and I felt that there was something important at stake until we could answer it. Recently, in [12] I showed that entanglement in such *physical* states is also typical, featuring a volume law. In this sense, we can say that the quantum approach gives firm grounds to the foundations of statistical mechanics.

2.3 Emergence of Lorentz Symmetry and Analogues of Black Hole Physics

The beauty of condensed matter physics is that, with the words of Phil Anderson, "more is different". Collective behavior of many-body systems can feature properties that are not properties of its elementary constituents. Moreover, the microscopic physics can be messy and ugly, but the emergent behavior beautiful and elegant from the mathematical point of view. A beautiful line of research is to show that a condensed matter system can feature excitations that behave just like elementary particles. For instance, fermions can emerge from topological order. The mechanism that creates topological order is that of the condensation of extended objects like closed strings and membranes. Such systems feature excitations made by the boundary of open strings and membranes that can behave like fermions [1, 24].
In his seminal work [11], X.-G. Wen showed that light can emerge in a system of screened dipoles. Thinking that photons are not elementary particles but collective modes of an underlying condensed matter system is a point of view rich of consequences. In my work [16], I have shown that in such system the speed of light *is* the maximum possible speed of signals. What one usually imposes as postulate, is here emergent. We have an emergent Lorentz symmetry. The result was obtained using the powerful machinery of the Lieb-Robinson bounds.
In the view of the principle of emergence, even gravity could be viewed as an emergent phenomenon. One could take the emergent point of view very seriously and think that the notion itself of locality and manifold can emerge from a condensed matter system. Recently, I have showed a Bose-Hubbard model where the particles do not hop on a fixed graph, but the graph itself is made of quantum degrees of freedom that interact with the particles [2, 3]. Moreover, also black hole physics can emerge from such many body toy models. From the condensed matter point of view, this Hubbard model with evolving graph contains a rich phase diagram, whose investigation will be the object of future research.

2.4 Quantum Phase Transitions, the Adiabatic Theorem, Lieb Robinson bounds and other topics

One of my interests of research is the use of novel tools to investigate universal properties of quantum critical points. I have been very interested in understanding how the critical points are revealed by geometric properties of the Hilbert space. In [22], I have shown that quantum critical points correspond to topological properties of the Berry phase and divergences in the adiabatic curvature. This work has influenced a series of highly cited works in which phase boundaries and quantum critical points are described by a differential geometric setup [18]. This approach can be used to obtain quantum speed-ups in the adiabatic preparation of quantum states or in protocols of adiabatic quantum computation, as I showed in [9], in which the problem of finding the fastest adiabatic path in the parameter space is mapped into a geodesic problem, see Fig.2.

The concept of adiabatic continuity to define the concept of phase goes to P. Anderson. In order to use such concept rigorously, one needs versions of adiabatic theorems that are suited for a system with a scaling number of degrees of freedom. I contributed to this topic with my paper [15] on the adiabatic theorem for many body systems. Moreover, using the adiabatic theorem, I showed an optimal way of preparing topologically ordered states [21].

The celebrated Lieb-Robinson bounds are one of the most powerful tools to study the correlations in quantum many body systems. These bounds mean that if a many body system is ruled by a local Hamiltonian, all the physics has local properties, even if we do not impose a relativistic structure, like in ordinary quantum mechanics. Thanks to this technique, it has been possible to prove the exponential decay of correlations in gapped systems and the stability of the gap in topologically ordered systems. Unfortunately, the Lieb-Robinson bounds are only valid in finite dimensional systems. In [27], I have proven a new theorem that extends the Lieb-Robinson result to a class of commutator bounded systems in an infinite-dimensional Hilbert space.

Recently, I have been studying QPTs in some one dimensional models, to understand phenomena like symmetry protection of topological order and factorization [29].

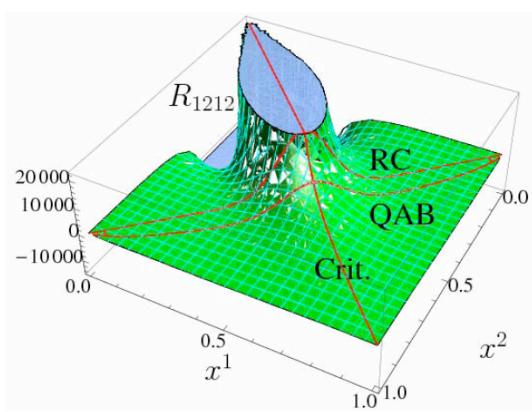

Figure 2: The curvature defined by the adiabatic condition and the geodesic path QAB that minimizes the time of a computation

3 Future Work

In the immediate future, I intend to expand some of the research lines that I described above:

- **Entanglement and Thermalization in physical states**

  There are many things to do. First and foremost, we need to implement energy constraints in the ensemble of physical states so to prove that the reduced system is typically a thermal state. This result would somehow complete the program of giving solid foundations to quantum statistical mechanics. I am currently also computing the converge rate to thermalization by means of gap analysis. An intriguing possibility is to use this formalism to prove that black holes are fast scramblers of information and thus the generality of certain aspects of black hole physics. Third, we need to be able to compute the von Neumann entropy. I am thrilled by the possibility of using this formalism to study the stability of topological phases. Since this formalism depends explicitly on the graph on which the system is defined, I think that it is just natural to inquire how we can use these tools to obtain graph-theoretic results.

- **Stability and Characterization of Topological Phases**

  In my view, these are some of the most intriguing open problems in theoretical condensed matter. How do we know whether topological states are really stable? Are their amazing features really the properties of a whole phase? Is topological entropy stable against perturbations and finite temperature? What is the relationship between the lifetime of quantum memory and topological entropy? What is the correct mathematical description for topological metastable states? Is the thermodynamic limit adequate to describe topological order? Can the behavior away from equilibrium serve to characterize topological phases? My current effort is in the investigation of these questions, and will certainly cover at least a couple of years (if not more, new questions arise all the time!).

- **Emergence of Symmetries** As we have seen [16], the Lieb-Robinson bound imply that models with emergent light force its speed to be the maximum speed of signals. This is a strong argument in favor of the emergence of Lorentz symmetry. In order to be a truly emergent phenomenon, this speed limit must universal and robust against perturbations. I want to show that the universality of the limit speed is protected at low energies by the same topological mechanism that protects the emergent photon.

In the so not immediate future, well, it is always difficult to say. But together with strongly correlated quantum systems, in the long term I would also like to understand complex systems like complex networks, macroeconomics, and biological systems.

- The classical version of the toy model of [2] can be used to describe the evolution of complex network like the world wide web. Moreover, the study of entanglement in physical states captures some of the graph theoretic structure underlying a local model. In the future, I am definitely going to spend more time in this line of research.

- Perhaps one of my greatest interests outside physics is economics. I have always been unsatisfied with the agent based models that try to describe economic behavior. We are not able to find out the behavior of materials from first principles, let alone behavior of human masses, which is far from being rational! Nevertheless, I believe that some tools of theoretical physics can still be used to gain insight in the economic behavior, like ideas coming from the renormalization group. Economic or social behavior have some intrinsic robustness (or criticality) that come from the fact that there is an underlying principle of organization. Stability of money, for instance, perhaps is the stability of a phase. I am looking forward to devoting part of my research to this topic.

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