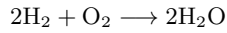


Resources and their use and consumption form a central part of our life. Many branches of science and engineering are concerned with the question of which given resource objects can be converted into which target resource objects. For example, information theory studies the conversion of a noisy communication channel instance into an exchange of information. Inspired by work in quantum information theory, we develop a general mathematical toolbox for this type of question. The convertibility of resources into other ones and the possibility of combining resources is accurately captured by the mathematics of ordered commutative monoids. As an intuitive example, we consider chemistry, where chemical reaction equations such as



are concerned both with a convertibility relation “ \longrightarrow ” and a combination operation “ $+$ ”. We study ordered commutative monoids from an algebraic and functional-analytic perspective and derive a wealth of results which should have applications to concrete resource theories, such as a formula for rates of conversion. As a running example showing that ordered commutative monoids are also of purely mathematical interest without the resource-theoretic interpretation, we exemplify our results with the ordered commutative monoid of graphs.

While closely related to both Girard’s linear logic and to Deutsch’s constructor theory, our framework also produces results very reminiscent of the utility theorem of von Neumann and Morgenstern in decision theory and of a theorem of Lieb and Yngvason on the foundations of thermodynamics.

Concerning pure algebra, our observation is that some pieces of algebra can be developed in a context in which equality is not necessarily symmetric, i.e. in which the equality relation is replaced by an ordering relation. For example, notions like cancellativity or torsion-freeness are still sensible and very natural concepts in our ordered setting.