# CO 639 Scribe Notes

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**Threshold Theorem**: $\exists$ threshold $p_c$ such that if error rate per physical gate/timestep $p < p_c$, then we can perform an arbitrary quantum computation using poly($\log z$) overhead per logical gate/timestep with error rate $\epsilon$ per logical gate/timestep (Depth = number of timesteps).

**Basic idea**: Concatenated codes:
1 level $p \to Cp^2$
2 levels $\to C(Cp^2)^2 = C^3p^4$
$\vdots$
$l$ levels $\to \frac{(Cp)^{2^l}}{C}$

For convergence, we want $Cp < 1$.

Set $p_c = \frac{1}{C}$ level $l$ error probability is $p_c(\frac{p}{p_c})^{2^l}$

Goals: Make this rigourous, and set lower bound on $p_c$

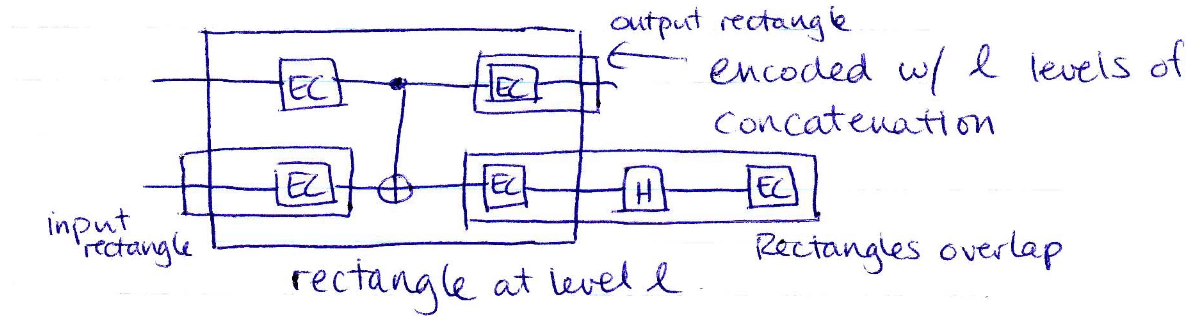
**Error model**:
Probabilistic error model:
Error with probability $p$, no error with probability $1 - p$
Adversarial choice of error type

Uncorrelated error:
i.e. independent error probability between qubits/timesteps (except when a gate interacts with two or more qubits)

prob($r$ errors)$\sim p^r$

**Additional assumptions**:
Parallel gates
Long-range gates between any pair of qubits
No leakage/erasure errors
Fresh ancillas

Measurements and classical comptuation in 1 time step
Measurements can have errors
Classical computation is reliable

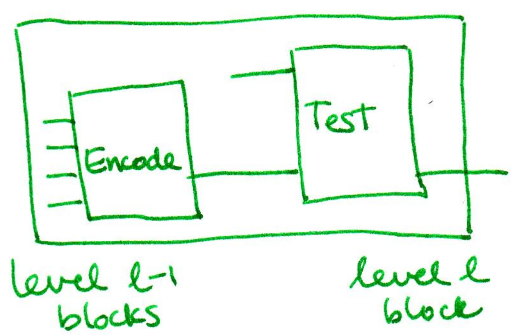



Each gate at level $l$ is followed by level $l-1$ error correction

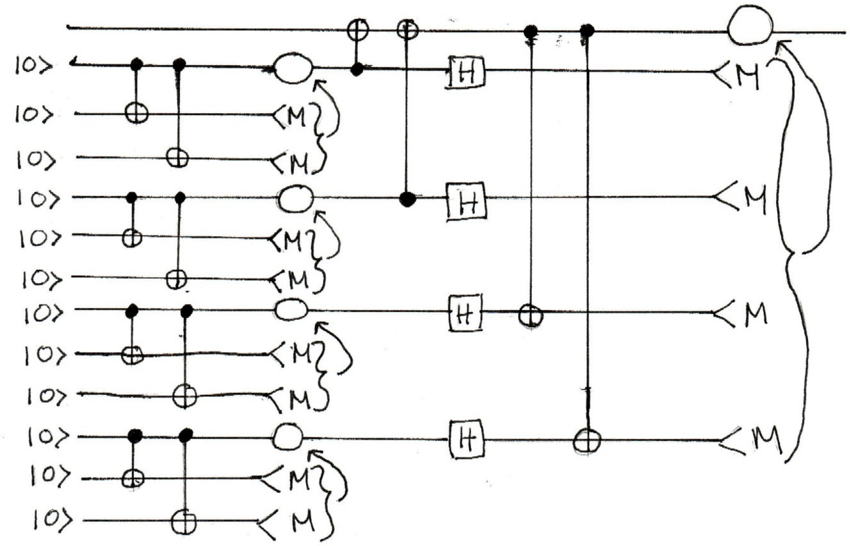
Physical qubits = level 0. Block of $7^l$ qubits = level $l$ (Assume [[7, 1, 3]])

Input rectangle for one big rectangle is output for previous big rectangle

Ancilla preparation rectangle:




Steane EC:




∗ If bit of phase syndromes agree, correct error. Otherwise, do nothing.

Note only 6 timesteps after ancilla preparation.

Level 0 gate is bad if it has a (new) error. Level 0 rectangle = level 0 gate.

Level $l$ rectangle is bad if $\geq 2$ of its level $l-1$ rectangles are bad. Good = not bad.

Block at level $l$ ($7^l$ qubits) is bad:
if it has experienced a bad level $l$ rectangle
interacted with a bad block
if it has 2 bad level $l-1$ subblocks

EC at level $l$ can convert bad level $l-1$ blocks to good level $l-1$ blocks:
EC contains <u>no</u> bad level $l-1$ rectangles $\Rightarrow$ all output level $l-1$ blocks are good
EC contains 1 bad rectangle, input state has 0 bad level $l-1$ blocks, output state has 1 bad level $l-1$ block (in appropriate place).