Book Review
on

Statistical Structure of Quantum Theory
by Alexander S. Holevo
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Some peg the beginning of quantum information theory at the year 1984, when the BB84 quantum cryptographic protocol was introduced. Others peg it at 1993, with the fantastic invention of quantum teleportation. A not wholly to be discounted view, however, is that quantum information actually took its start in the years 1972-73. In that period A. S. Holevo published three seminal papers—one having to do with why we must consider the positive operator-valued measures among the fleet of possible measurements used for decoding a quantum signal; one having to do with the very definition of a quantum mechanical channel as a linear completely-positive trace-preserving map; and one having to do with the capacity of a quantum channel for conveying classical information. It is thus a pleasure to see, now almost 30 years later, these results and many more from the wider community tied together in a single book whose aim is to turn inward and ask, "What might all of this tell us about the statistical structure upon which quantum theory is grounded?"

There is a lot of information squeezed into the scant 159 pages of this book, and it should be treated as a survey more than anything. The philosophy that guides the presentation is that though quantum theory does have a dynamical aspect, a predominant part of the theory is simply about a new kind of probabilistic model for expressing uncertainties of various kinds. It is a probabilistic model that differs radically in character from the classical model first laid down by A. N. Kolmogorov in 1933 as "measure theory with a soul."

After a 10 page survey of its contents, the book sets about its job in five roughly equal-sized chapters. The first captures what might be termed "old school" quantum mechanics: treating observables as Hermitian operators, the Born rule, uncertainty principles and commutation relations, tensor products, no-go theorems for hidden variables, and symmetry operations in quantum-state spaces. It is in the second chapter that the book starts to come into its own. Here, for instance, one sees material only now starting to be covered in textbooks because of the influence of quantum information theory.

Chapters 2 and 3, "Statistics of Quantum Measurements" and "Evolution of an Open System," get us to the theory of positive-operator valued measures (POVMs) and completely-positive maps, and most importantly tell us what they are good for. In Chapter 2, we see some of the theory of quantum detection at work: Bayes risk functions, Cramér-Rao inequalities, logarithmic derivatives, geometries for quantum states, superadditivity of the transmission capacity of noncommuting quantum states, and group covariant observables—

that is, POVMs whose elements are related to each other via the action of elements drawn from a unitary representation of some group. In Chapter 3, we find first a discussion of various general properties for completely positive maps—for instance that they obey a kind of operator Schwarz inequality, and that the quantum relative entropy is monotone under their action. It then caps off with a discussion of the various kinds of capacity a quantum channel can have and a detailed exposition of quantum dynamical semigroups.

Chapters 4 and 5, "Repeated and Continuous Measurement Processes" and "Processes in Fock Space," are in a sense more advanced (at least for this reader), relying more heavily on a familiarity with measure theory for their digestion. First the concept of a completely positive instrument is introduced. Basically this is the theory of general quantum-state changes associated with measurement processes, including measurements with a continuous number of outcomes and measurements continuous in time. From there, there are discussions of the repeatability of generalized measurements, the standard quantum limit and its transcendence with clever measurement interactions, quantum nondemolition measurements, and the quantum Zeno effect. More important, but less well known to this community, are the limit theorem for convolutions of instruments (crucial for the understanding of continuous measurement processes), facts on convolution semigroups of instruments, and instrumental processes. The closing chapter focuses on quantum stochastic calculus and purification theorems (here called dilation theorems) for giving unitary dynamical descriptions of instrumental processes. Subtopics include the quantum Ito formula, quantum stochastic differential equations, and Wiener and Poisson processes in Fock space.

All the lectures in this book are presented in the author's characteristic no-nonsense style, making it a gem of a reference work. The bibliography, in fact, is a rich source of material, indispensable for filling in how to actually derive all the advertised results. The clear-cut purpose of the monograph is not to go through the mechanics of each theorem, but to paint a picture in modern art. Throughout, there is an emphasis on how each and every quantum result differs from its analog in the classical probabilistic model. One does, however, fear that a central illuminating thread is somehow missing from the exposition, though the author cannot be blamed: It is a thread no one has yet seen. A radical departure from classical probability theory, yes. But why? What is the all-important, undeniable physical fact that forces this revised calculus upon us? Can we state it in a way that does not consist of a dozen disparate theorems? One day we will get there. In the mean time, though, an exposition like this is part of the necessary journey. It is likely to be as Gertrude Stein said of the more classical kind of modern art, "It looks strange and it looks strange and it looks very strange; and then suddenly it doesn't look strange at all and you can't understand what made it look strange in the first place."

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